

A classical optical approach to the ‘non-local Pancharatnam-like phases’ in Hanbury-Brown-Twiss correlations

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We examine a recent proposal to show the presence of nonlocal Pancharatnam type geometric phases in a quantum mechanical treatment of intensity interferometry measurements upon inclusion of polarizing elements in the setup. It is shown that a completely classical statistical treatment of such effects is adequate for practical purposes. Further we show that the phase angles that appear in the correlations, while at first sight appearing to resemble Pancharatnam phases in their mathematical structure, cannot actually be interpreted in that manner. We also describe a simpler Mach-Zehnder type setup where similar effects can be observed without use of the paraxial approximation.

I. INTRODUCTION

The work of Hanbury-Brown and Twiss (HBT) about six decades ago inaugurating the field of intensity interferometry in radio astronomy as well as in the visible region constituted a major conceptual and experimental advance in the subject [1]. Even though initially there was some confusion regarding interpretation, especially with regard to the quantum mechanical meaning of HBT correlations, it has since been recognized that it can be satisfactorily understood in terms of the statistical features of general states of classical (optical) wave fields. Each such statistical state is describable by a hierarchy of correlation functions of various orders, and the HBT intensity-intensity correlation function stands one step beyond the more familiar Young type amplitude-amplitude correlation function (also called the two-point function describing partial coherence) adequate for handling interference and diffraction phenomena. As with the Bell inequalities which characterize proposed local realist extensions of quantum mechanics, and which can be violated by specific entangled quantum states, in the HBT case too particular quantum states of radiation may lead to correlations beyond what classical theory can explain. However this does not invalidate the fact that as a concept the HBT correlations are classically meaningful.

In contrast to the HBT effect, the concept of geometric phases in quantum mechanics was uncovered by Berry just over three decades ago [2]. His analysis was in the

framework of adiabatic cyclic unitary evolution of pure quantum states obeying the time-dependent Schrödinger equation - at the end of such evolution the state vector (or wave function) in Hilbert space acquires a new previously unrecognized phase. Later rapid developments greatly clarified the situation - the geometric phase is (in the language of quantum mechanics) a ray space quantity; it can be defined even in nonadiabatic and noncyclic evolutions [3–5]; and it is meaningful in purely classical wave optical situations, so it is not specifically quantum mechanical in origin. Indeed it was soon realised that a phase found by Pancharatnam in 1956 in classical polarization optics was an early precursor of the geometric phase in a nonadiabatic cyclic situation, with the Poincaré sphere of polarization optics playing the role of ray space in quantum mechanics [6–8].

Subsequent work on the kinematic approach to the geometric phase has shown that the basic ingredient is the use of a complex Hilbert space to describe (pure) states of a physical system, whether in quantum mechanics or in classical wave optics, and the associated ray space [9, 10]. It has also brought out the relevance of the Bargmann invariants for geometric phase theory [11]. While some attempts have been made to define geometric phases for mixed state evolution [12–15], the emphasis at the basic level has been on pure states.

Against this background, some very interesting recent work has attempted to bring together these two independent developments in an unexpected manner [16]. It has been shown that in a carefully prepared experimental setup involving polarizing gadgets the expression for HBT correlations contains just the kind of phase angle - a solid angle on a two-sphere - involved in Pancharatnam’s work. This has been described as a nonlocal form of the

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Pancharatnam phase, and it has been subjected to an experimental test as well [17, 18]. In particular the theoretical analysis uses the photon description of light, involving quantized radiation field operators; and the nonlocal effect has been characterized as a genuinely two-photon property not visible at the single photon level.

The aims of the present work are two-fold : the first is to show that a purely classical statistical treatment of radiation is adequate to obtain the result of [16], without having to use the photon picture based on the quantum theory of radiation; the second is to significantly simplify the experimental set up while retaining the appearance of the Pancharatnam solid angle in the expression for HBT correlations, and so to understand better whether it is indeed a nonlocal form of the Pancharatnam phase which in any event is a particular instance of the geometric phase.

The contents of the paper are arranged as follows: Section II describes the original HBT scheme of [16] from a purely classical point of view. In Section III we give a scheme based on a Mach-Zender interferometer, where similar results are obtained without the paraxial approximation. Section IV offers some concluding remarks and the Appendix describes the mathematical conventions used in this paper.

II. CLASSICAL TREATMENT OF HBT CORRELATIONS WITH POLARIZERS

In this Section we present a completely classical treatment of the HBT correlations involving the Pancharatnam solid angle, comparing with [16] at relevant points.

We assume two localized sources S_1, S_2 of quasi-monochromatic radiation of mean frequency ω_0 , in independent statistical states (more fully specified below). As in Fig. 1, they are located a distance s apart, at points $\mathbf{x}_1, \mathbf{x}_2$ along the x -axis of a spatial coordinate system. Light from each source reaches two detectors D_3 and D_4 at positions $\mathbf{x}_3, \mathbf{x}_4$ a distance d apart, and at a distance l from the sources in the overall direction of the positive z -axis, with $l \gg s, d$. Therefore the propagation vectors of light waves from S_1, S_2 to D_3, D_4 may all be treated as practically parallel and along the positive z -axis, i.e. we are in the paraxial regime. Polarizers P_R, P_L placed immediately after S_1, S_2 select right and left circular polarizations respectively. Just before reaching detector D_3 , the superposed fields from S_1 and S_2 pass through a linear polarizer $P(\theta_3)$ at an angle θ_3 in the transverse x - y plane; similarly another linear polarizer $P(\theta_4)$ at angle θ_4 is placed just before D_4 . Within the limits of the paraxial approximation all relevant electric field vectors can be taken to be two-component objects in the common transverse plane, with only x and y components.

In the absence of polarizers $P_R, P_L, P(\theta_3), P(\theta_4)$, using the Kirchoff and paraxial approximations the positive frequency analytic signal electric field vectors reaching D_3 from $S_a, a = 1, 2$, are given by

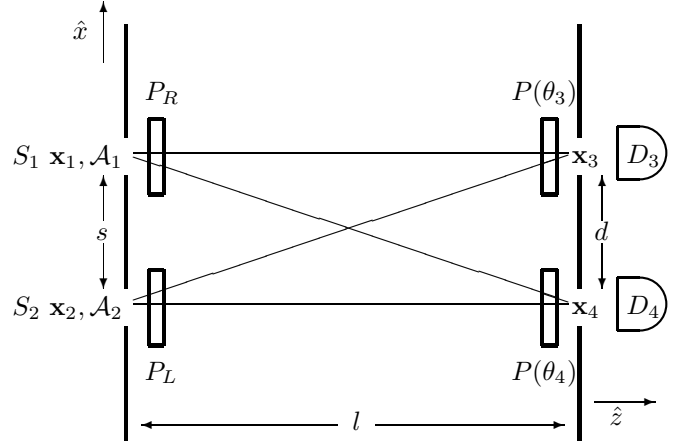


FIG. 1. Scheme for HBT correlations with polarizers.

$$\begin{aligned} \underline{E}^{(a)}(\mathbf{x}_3, t) &\approx u_{a3} \underline{E}^{(a)}\left(\mathbf{x}_a, t - \frac{\mathcal{R}_{a3}}{c}\right), \\ u_{a3} &= \frac{-ik_0 l}{2\pi} \frac{\mathcal{A}_a}{\mathcal{R}_{a3}^2}, \mathcal{R}_{a3} = |\mathbf{x}_a - \mathbf{x}_3|, k_0 = \frac{\omega_0}{c}, a = 1, 2 \end{aligned} \quad (1)$$

Here \mathcal{A}_a is the effective area of the source S_a , u_{a3} is a dimensionless geometrical propagation factor from S_a to D_3 , and $\underline{E}^{(a)}(\mathbf{x}_a, t - \mathcal{R}_{a3}/c)$ is the electric field vector at S_a at the retarded time. We treat this field as effectively constant over \mathcal{A}_a . With the polarizers in place as in Fig. 1, the total field reaching D_3 is

$$\begin{aligned} \underline{E}(\mathbf{x}_3, t) &\approx P(\theta_3) \left(u_{13} P_R \underline{E}^{(1)}(\mathbf{x}_1, t - \mathcal{R}_{13}/c) \right. \\ &\quad \left. + u_{23} P_L \underline{E}^{(2)}(\mathbf{x}_2, t - \mathcal{R}_{23}/c) \right). \end{aligned} \quad (2)$$

For $\underline{E}(\mathbf{x}_4, t)$ at D_4 we have a similar expression by replacing $\mathbf{x}_3 \rightarrow \mathbf{x}_4$, $u_{a3} \rightarrow u_{a4}$, $\mathcal{R}_{a3} \rightarrow \mathcal{R}_{a4}$, $P(\theta_3) \rightarrow P(\theta_4)$.

The intensities at D_3, D_4 are proportional to $\underline{E}^*(\mathbf{x}_3, t) \cdot \underline{E}(\mathbf{x}_3, t)$ and $\underline{E}^*(\mathbf{x}_4, t) \cdot \underline{E}(\mathbf{x}_4, t)$ respectively. Since we are concerned with a completely classical discussion, it is appropriate to treat the fields $\underline{E}^{(a)}(\mathbf{x}_a, t) \equiv \underline{E}^{(a)}$ as belonging to corresponding statistical ensembles, described by appropriate classical probability distributions. For our purposes these are represented by correlation functions of various orders [19, 20]. (The time arguments will be seen to be irrelevant, and as $l \gg s, d$ the differences in retarded times will be neglected). Denoting ensemble averages by $\langle \dots \rangle$, the HBT correlation of intensities at D_3 and D_4 is proportional to

$$\begin{aligned} \Gamma^{(2,2)} &= \langle \underline{E}^*(\mathbf{x}_3, t) \cdot \underline{E}(\mathbf{x}_3, t) \underline{E}^*(\mathbf{x}_4, t) \cdot \underline{E}(\mathbf{x}_4, t) \rangle = \\ &\langle (u_{13}^* \underline{E}^{(1)\dagger} P_R + u_{23}^* \underline{E}^{(2)\dagger} P_L) P(\theta_3) (u_{13} P_R \underline{E}^{(1)} + u_{23} P_L \underline{E}^{(2)}) \cdot \\ &\quad (u_{14}^* \underline{E}^{(1)\dagger} P_R + u_{24}^* \underline{E}^{(2)\dagger} P_L) P(\theta_4) (u_{14} P_R \underline{E}^{(1)} + u_{24} P_L \underline{E}^{(2)}) \rangle \end{aligned} \quad (3)$$

This is a sum of sixteen terms, each a product of four factors, which can be labelled in sequence as

1111, 1112, \dots , 2221, 2222 according to which term from each of the four bracketed expressions is included in the product. Thus for example

$$\begin{aligned} 1111 &= |u_{13}|^2 |u_{14}|^2. \\ \langle \underline{E}^{(1)\dagger} P_R P(\theta_3) P_R \underline{E}^{(1)} \underline{E}^{(1)\dagger} P_R P(\theta_4) P_R \underline{E}^{(1)} \rangle, \\ 1112 &= |u_{13}|^2 u_{14}^* u_{24}. \\ \langle \underline{E}^{(1)\dagger} P_R P(\theta_3) P_R \underline{E}^{(1)} \underline{E}^{(1)\dagger} P_R P(\theta_4) P_L \underline{E}^{(2)} \rangle. \end{aligned} \quad (4)$$

We now specify the statistics of the source fields $\underline{E}^{(a)}$. We make the physically plausible assumption that $\underline{E}^{(1)}$ and $\underline{E}^{(2)}$ belong to two independent time stationary centered random phase unpolarized Gaussian ensembles. Recalling that $\underline{E}^{(1)}$, $\underline{E}^{(2)}$ are two-component complex column vectors with x and y components labelled by $\alpha, \beta = 1, 2$, we have the basic ensemble averages

$$\begin{aligned} \langle \underline{E}_\alpha^{(a)} \rangle &= \langle \underline{E}_\alpha^{(a)*} \rangle = 0, \\ \langle \underline{E}_\alpha^{(a)} \underline{E}_\beta^{(a)} \rangle &= \langle \underline{E}_\alpha^{(a)*} \underline{E}_\beta^{(a)*} \rangle = 0, \quad a = 1, 2; \\ \langle \underline{E}_\alpha^{(1)*} \underline{E}_\beta^{(1)} \rangle &= \kappa \delta_{\alpha\beta}, \quad \langle \underline{E}_\alpha^{(2)*} \underline{E}_\beta^{(2)} \rangle = \kappa' \delta_{\alpha\beta}; \end{aligned} \quad (5)$$

and the derived averages

$$\begin{aligned} \langle \underline{E}_{\alpha'}^{(1)*} \underline{E}_{\beta'}^{(1)*} \underline{E}_\alpha^{(1)} \underline{E}_\beta^{(1)} \rangle &= \kappa^2 (\delta_{\alpha'\alpha} \delta_{\beta'\beta} + \delta_{\alpha'\beta} \delta_{\beta'\alpha}), \\ \langle \underline{E}_{\alpha'}^{(2)*} \underline{E}_{\beta'}^{(2)*} \underline{E}_\alpha^{(2)} \underline{E}_\beta^{(2)} \rangle &= \kappa'^2 (\delta_{\alpha'\alpha} \delta_{\beta'\beta} + \delta_{\alpha'\beta} \delta_{\beta'\alpha}). \end{aligned} \quad (6)$$

Here κ, κ' are in general different real positive parameters. These expressions can be easily reproduced by a suitable centered Gaussian probability distribution for the four complex amplitudes $\{\underline{E}_\alpha^{(a)}\}$ treating all polarizations uniformly.

With these assumptions on the sources S_1, S_2 there are only six nonvanishing terms in $\Gamma^{(2,2)}$ corresponding to the products 1111, 1122, 1221, 2112, 2211 and 2222. These can be easily computed using the basic properties of the 2×2 polarization matrices $P_R, P_L, P(\theta_3), P(\theta_4)$ given in the Appendix. We find:

$$\begin{aligned} 1111 &= |u_{13}|^2 |u_{14}|^2 \frac{\kappa^2}{2}; \\ 1122 &= |u_{13}|^2 |u_{24}|^2 \frac{\kappa \kappa'}{4}; \\ 1221 &= u_{13}^* u_{23} u_{24}^* u_{14} \kappa \kappa' \text{Tr}(P_R P(\theta_3) P_L(\theta_4)); \\ 2112 &= (1221)^*; \\ 2211 &= |u_{23}|^2 |u_{14}|^2 \frac{\kappa \kappa'}{4}; \\ 2222 &= |u_{23}|^2 |u_{24}|^2 \frac{\kappa'^2}{2}; \\ \Gamma^{(2,2)} &= \frac{1}{2} (|u_{13} u_{14}|^2 \kappa^2 + |u_{23} u_{24}|^2 \kappa'^2) \\ &\quad + \frac{1}{4} (|u_{13} u_{24}|^2 + |u_{14} u_{23}|^2) \kappa \kappa' \\ &\quad + 2 \kappa \kappa' \text{Re} [u_{13}^* u_{23} u_{14} u_{24}^* \text{Tr}(P_R P(\theta_3) P_L P(\theta_4))]. \end{aligned} \quad (7)$$

The last term in $\Gamma^{(2,2)}$ is reminiscent of the Pancharatnam phase in the geometric phase context. As shown in the Appendix, the trace term has a phase related to the solid angle of a ‘lune’ on a two-sphere enclosed by the two meridians at polar angles $2\theta_3$ and $2\theta_4$,

$$\text{Tr}(P_R P(\theta_3) P_L P(\theta_4)) = \frac{1}{4} e^{-2i(\theta_3 - \theta_4)}, \quad (8)$$

so after including the propagation factors u in Eqn.(1) the last term in $\Gamma^{(2,2)}$ becomes

$$\frac{\kappa \kappa'}{2} \left(\frac{k_0 l}{2\pi} \right)^4 (\mathcal{A}_1 \mathcal{A}_2 / \mathcal{R}_{13} \mathcal{R}_{23} \mathcal{R}_{14} \mathcal{R}_{24})^2 \cos 2(\theta_3 - \theta_4). \quad (9)$$

This is the contribution made by the presence of the polarizers to the HBT correlations in the setup of Fig. 1.

It is clear that the result (9) depends strongly on the statistical properties (5,6) assumed for the sources S_1 and S_2 . Even a slight change in them can lead to the final result for $\Gamma^{(2,2)}$ not being expressible in terms of any recognizable solid angle on a two-sphere at all. Thus the fact that the Pancharatnam solid angle has entered the result (9) seems to be not generic or robust.

We point out that the quantum mechanical treatment in [16] involves defining photon annihilation and creation operators $a_3, a_3^\dagger, a_4, a_4^\dagger$ corresponding to the modes ‘at the detectors’ D_3, D_4 in terms of operators $a_1, a_1^\dagger, a_2, a_2^\dagger$ for modes ‘at the sources’ S_1, S_2 . The proposed relationships are exactly parallel to Eqn. (2) above for classical analytic signals and read:

$$\begin{aligned} a_3 &= P(\theta_3)(u_{13} P_R a_1 + u_{23} P_L a_2), \\ a_4 &= P(\theta_4)(u_{14} P_R a_1 + u_{24} P_L a_2), \end{aligned} \quad (10)$$

and their adjoints for a_3^\dagger, a_4^\dagger . Here a_3 is a column vector of two annihilation operators for the two photon polarization states ‘at D_3 ’, a_3^\dagger is a row vector, etc.

However, using the classical wave propagation formulae for photon operators in this way cannot be expected to preserve the bosonic commutation relations, and the meaning of $\langle a_3^\dagger a_3 \rangle, \langle a_4^\dagger a_4 \rangle$ in terms of photon numbers. A proper quantum mechanical treatment needs to address these points in a satisfactory manner.

Nevertheless, the fact that the use of polarizers in HBT correlation measurements leads to nontrivial consequences, while not surprising, is interesting in itself. The proposed interpretation in the language of Pancharatnam(and geometric) phases is examined more closely in the next Section.

III. AN ALTERNATIVE EXPERIMENTAL SCHEME

The paraxial approximation used in the previous Section suggests that a common Poincaré sphere could be used to describe the polarization states of transverse electric fields propagating in any of the four practically parallel directions S_1, S_2 to D_3, D_4 . However it should also

be mentioned that the analysis did not require following the evolution of the polarization state of any electric field two-vector along any closed circuit on a Poincaré sphere, which is usually a prerequisite to identify a Pancharatnam phase.

To understand the situation better we now consider a simpler scheme involving only two mutually orthogonal propagation directions, thus giving up the paraxial property altogether. This is similar to but considerably simpler than the one discussed in [18]. We should point out that in general, when there are plane waves simultaneously propagating in several different directions, one should in principle use a separate Poincaré sphere attached to each propagation direction to follow the polarization state of the field vector propagating in that direction. Furthermore, for each propagation direction there is an independent $SO(2)$ freedom in the choice of transverse axes, and even a $U(2)$ freedom in the choice of orthonormal polarization states, in setting up these individual Poincaré spheres. It is only when there are physically significant simplifying features, such as the paraxial condition for instance, that many different Poincaré spheres may be identified with one another.

We consider a setup with two sources or input ports A, B producing plane electromagnetic waves of common frequency ω_0 , propagating along the positive z and x axes respectively, as in Fig. 2 (the y axis then points out of the page). These waves pass through circular polarizers P_R located close to the sources. The field from A is described by a two-component complex column vector \underline{E} made up of its x and y components, while \underline{E}' from B is similarly described by its y and z components:

$$\underline{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}, \quad \underline{E}' = \begin{pmatrix} E'_y \\ E'_z \end{pmatrix} \quad (11)$$

This is a natural convention for the present non-paraxial situation.

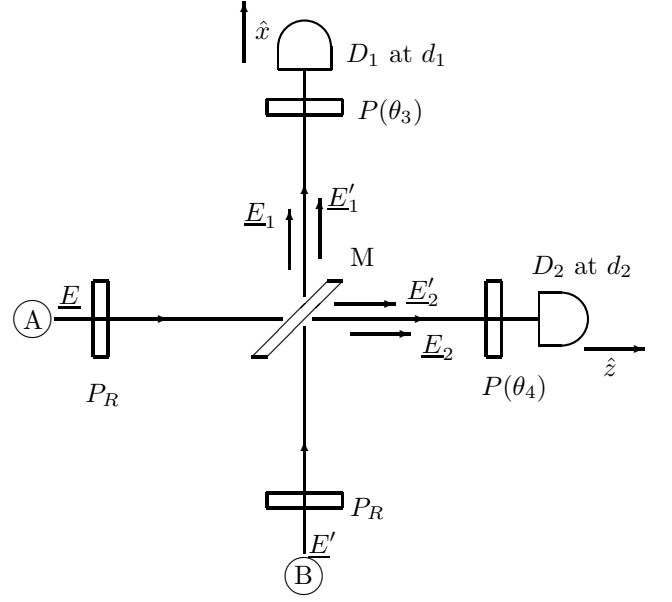


FIG. 2. A setup for HBT correlations in the nonparaxial condition.

The plane wave factors are $e^{ik_0 z}$ for \underline{E} and $e^{ik_0 x}$ for \underline{E}' , but in the sequel these can be omitted.

After the polarizers P_R the two plane waves arrive at a half-silvered plane mirror M placed at an angle of 45° bisecting the z and x axes. Each plane wave is partially transmitted and partially reflected by M , the intensities divided equally. Thus \underline{E} gives rise to reflected \underline{E}_1 and transmitted \underline{E}_2 , while \underline{E}' gives rise to transmitted \underline{E}'_1 and reflected \underline{E}'_2 . Both \underline{E}'_1 and \underline{E}_1 are described like \underline{E} in (11), \underline{E}_2 and \underline{E}'_2 like \underline{E} . At the mirror, the condition that there be no tangential field (and the stated reduction in intensity) determines \underline{E}_1 in terms of \underline{E} , and \underline{E}'_2 in terms of \underline{E}' . Keeping in mind the conventions (11), it is seen that mirror reflection is expressed by the matrix τ_2 (Pauli matrix σ_1 as explained in Eqn. (18)). Since we are dealing with circular polarizations, upon reflection RCP and LCP get interchanged, while upon transmission they each remain unchanged. All in all, the two sets of reflected and transmitted waves are:

$$\begin{aligned} \text{Reflected} \quad \underline{E}_1 &= -\frac{1}{\sqrt{2}}\tau_2 P_R \underline{E}, \quad \underline{E}'_2 = -\frac{1}{\sqrt{2}}\tau_2 P_R \underline{E}'; \\ \text{Transmitted} \quad \underline{E}_2 &= \frac{1}{\sqrt{2}}P_R \underline{E}, \quad \underline{E}'_1 = \frac{1}{\sqrt{2}}P_R \underline{E}'. \end{aligned} \quad (12)$$

The projection matrix P_R of Eq. (19) can be used consistently for both \underline{E} and \underline{E}' , using the conventions(11).

After superposition, $\underline{E}_1 + \underline{E}'_1$ passes through a linear polarizer $P(\theta_3)$ (in the $y-z$ plane), while $\underline{E}_2 + \underline{E}'_2$ passes through $P(\theta_4)$ (in the $x-y$ plane). Each of these superpositions consists of one RCP wave and one LCP wave. They are then received at detectors D_1, D_2 respectively, after having traversed distances d_1 and d_2 from the mir-

ror M . The total fields reaching the detectors are:

$$\begin{aligned}\underline{E}(D_1) &= \frac{1}{\sqrt{2}}P(\theta_3)(-\tau_2 P_R \underline{E} + P_R \underline{E}'), \\ \underline{E}(D_2) &= \frac{1}{\sqrt{2}}P(\theta_4)(P_R \underline{E} - \tau_2 P_R \underline{E}').\end{aligned}\quad (13)$$

Therefore the HBT correlation of the two intensities is

$$\begin{aligned}\Gamma^{(2,2)} &= \langle \underline{E}(D_1)^\dagger \underline{E}(D_1) \underline{E}(D_2)^\dagger \underline{E}(D_2) \rangle \\ &= \frac{1}{4} \langle (-\underline{E}^\dagger P_R \tau_2 + \underline{E}'^\dagger P_R) P(\theta_3) (-\tau_2 P_R \underline{E} + P_R \underline{E}') \\ &\quad (\underline{E}^\dagger P_R - \underline{E}'^\dagger P_R \tau_2) P(\theta_4) (P_R \underline{E} - \tau_2 P_R \underline{E}') \rangle.\end{aligned}\quad (14)$$

Let us assume the same statistical properties for $\underline{E}, \underline{E}'$ here as for $\underline{E}^{(1)}, \underline{E}^{(2)}$ in Section II: independent, time stationary, centred random phase unpolarized Gaussian ensembles. Then Eqns.(5,6) are again valid with $\underline{E}^{(1)}, \underline{E}^{(2)} \rightarrow \underline{E}, \underline{E}'$. In expression (14) there are again sixteen terms of which (as in Section II) only 1111, 1122, 1221, 2112, 2211 and 2222 are nonzero. These have the values:

$$\begin{aligned}1111 &= \frac{\kappa^2}{8}; \quad 1122 = \frac{\kappa\kappa'}{16}; \\ 1221 &= \frac{\kappa\kappa'}{4} \text{Tr}(P_R P(\pi/2 - \theta_3) P_L P(\theta_4)); \\ 2112 &= (1221)^*; \\ 2211 &= \frac{\kappa\kappa'}{16}; \quad 2222 = \frac{\kappa'^2}{8}.\end{aligned}\quad (15)$$

The final result for the HBT correlations is:

$$\begin{aligned}\text{Tr}(P_R P(\pi/2 - \theta_3) P_L P(\theta_4)) &= -\frac{1}{4}e^{2i(\theta_3 + \theta_4)}, \\ \Gamma^{(2,2)} &= \frac{1}{8}(\kappa^2 + \kappa\kappa' + \kappa'^2) - \frac{\kappa\kappa'}{8} \cos 2(\theta_3 + \theta_4).\end{aligned}\quad (16)$$

The similarity to the expressions and results in Section II is evident. However since now the two propagation directions are mutually perpendicular, there is no privileged Poincaré sphere in the problem. It is true that the phase of the trace in Eq. (16) is most simply viewed as the solid angle of a lune on a two-sphere, but this two-sphere is useful for calculational purposes alone and cannot be identified in any compelling manner with any physically meaningful Poincaré sphere in the present setup. For this reason it seems not possible to interpret the phase in eq(3.6) as a nonlocal Pancharatnam phase, indeed as a geometrical phase at all. Added to this is the fact that once again even a small change in the statistical properties of \underline{E} and \underline{E}' is likely to alter the result (16) completely in structure.

IV. CONCLUDING REMARKS

The attempt in [16] to bring together the ideas of HBT correlations and Pancharatnam - or more generally ge-

ometric - phases is a very appealing one, even though our analysis suggests that this has not been achieved. In discussions of quantum measurement theory, the appearance of nonlocality is in connection with composite systems in entangled states of spatially separated subsystems and observations on them. Since in our classical treatment we obtain the same expression for HBT correlations as found in [16], it is clear that there is no quantum nonlocality involved. Further the way in which the Pancharatnam-like solid angle appears here does not depend on transporting a pure state of any system over a closed path in any parameter or state space; hence while it reveals an interesting feature of HBT correlations which have indeed been experimentally verified [17], its interpretation as a Pancharatnam phase seems open to question.

In our view, from a wider perspective, the situation under discussion has similarities to the van Cittert-Zernike theorem in partial coherence theory - even with a spatially incoherent source, propagation can produce partial coherence at the amplitude level. In a similar manner, in the case of HBT correlations too we see that even if the source intensities are uncorrelated, after propagation nontrivial HBT correlations can develop, since each detector (as in Figures 1 and 2) receives inputs from each source. An element of nonlocality here is quite natural since wave propagation involves spreading in space. If polarizers are placed on the paths of propagating beams, the fact that they can influence the HBT correlations is also quite natural. However for this to be interpretable in geometric phase terms it seems necessary to have an underlying complex Hilbert space structure and the pure state evolution concept in a physically significant manner. This feature is absent in the experimental setups and theoretical analysis described in Sections II and III.

While we have shown that an entirely classical treatment suffices to obtain the results of [16] there are states of radiation which are genuinely non classical and for which HBT correlations take values beyond the classical range of possibilities. Such states can certainly be used in the experimental setups of Sections II and III. A proper quantum mechanical treatment capable of handling such states must respect the fundamental commutation relations describing photons. This analysis will be presented elsewhere.

APPENDIX

We collect here some elementary formulae concerning 2×2 polarization matrices needed in the text. The matrices $P_R, P_L, P(\theta)$ are Hermitian projections onto corre-

sponding two-component column vectors:

$$\begin{aligned} |R\rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, |L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}, |\theta\rangle = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}; \\ \langle R, L | \theta \rangle &= \frac{1}{\sqrt{2}} e^{\mp i\theta}; \\ P_R &= |R\rangle\langle R|, \quad P_L = |L\rangle\langle L|, \quad P(\theta) = P(\theta + \pi) = |\theta\rangle\langle\theta|. \end{aligned} \quad (17)$$

The $\underline{\tau}$ matrices of polarization optics are a cyclic rearrangement of the Pauli matrices $\underline{\sigma}$ of quantum mechanics:

$$\tau_1 = \sigma_3, \quad \tau_2 = \sigma_1, \quad \tau_3 = \sigma_2. \quad (18)$$

In terms of them we have

$$\begin{aligned} P_R &= \frac{1}{2}(1 + \tau_3), \quad P_L = \frac{1}{2}(1 - \tau_3), \\ P(\theta) &= \frac{1}{2}(1 + \tau_1 \cos 2\theta + \tau_2 \sin 2\theta). \end{aligned} \quad (19)$$

Thus the circular polarization projectors P_R, P_L correspond to the N and S poles, $(0, 0, \pm 1)$, on the Poincaré sphere; while the linear polarization projector $P(\theta)$ maps to $(\cos 2\theta, \sin 2\theta, 0)$ on the equator.

The elementary traces needed in Eqn.(7) are

$$\text{Tr}(P_R P(\theta)) = \text{Tr}(P_L P(\theta)) = |\langle R, L | \theta \rangle|^2 = \frac{1}{2}. \quad (20)$$

The nontrivial trace in Eqn.(7) is a complex quantity:

$$\begin{aligned} \text{Tr}(P_R P(\theta_3) P_L P(\theta_4)) &= \frac{1}{4} \exp(i \arg \Delta_4(|R\rangle, |\theta_3\rangle, |L\rangle, |\theta_4\rangle)), \\ \Delta_4(|R\rangle, |\theta_3\rangle, |L\rangle, |\theta_4\rangle) &= \langle R | \theta_3 \rangle \langle \theta_3 | L \rangle \langle L | \theta_4 \rangle \langle \theta_4 | R \rangle. \end{aligned} \quad (21)$$

This Δ_4 is a four-vertex Bargmann invariant, whose phase is known to be (the negative of) a geometric phase. More precisely, $\arg \Delta_4(|R\rangle, |\theta_3\rangle, |L\rangle, |\theta_4\rangle) = -\frac{1}{2}\Omega$, where Ω is the solid angle on the Poincaré sphere enclosed by the meridian from N to S at polar angle $2\theta_3$ followed by the meridian from S to N at polar angle $2\theta_4$:

$$\begin{aligned} \text{Tr}(P_R P(\theta_3) P_L P(\theta_4)) &= \frac{1}{4} e^{-i\Omega/2}, \\ \Omega &= 4(\theta_3 - \theta_4). \end{aligned} \quad (22)$$

In the scheme of Fig. 2 in Section III, reflection at a mirror amounts to action on two-component electric field vectors $\underline{E}, \underline{E}'$ by the matrix $-\tau_2$. With respect to the polarization matrices the properties of τ_2 are:

$$\tau_2 P_R \tau_2 = P_L, \tau_2 P_L \tau_2 = P_R, \tau_2 P(\theta) \tau_2 = P(\pi/2 - \theta). \quad (23)$$

These are relevant in connection with Eqns. (15,16).

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